18.100C Homework 2
Due online by noon on Monday, September 28

This assignment has four parts. All four parts should be submitted in .tex and
.pdf formats through the Stellar course website. Parts 1-3 should be in a single
document and Part 4 in a separate document.

(1) For each of the following statements, write down the inverse, converse and
ccontrapositive. Indicate which of the statements are logically equivalent to
each other due to a simple rule of logic. State (but you need not prove)
which statements are true and which are false.
(a) If \( r + s \) is rational then \( r \) is rational or \( s \) is irrational.
(b) If \( r \) is irrational or \( s \) is irrational then \( r + s \) is irrational.
(c) If \( A \) is open then \( A^C \) is closed. (Take as given that \( A \) is in the universe
of subsets of a given metric space \( X \).)
(d) If \( 0 < x < y \) then \( 0 < \frac{1}{y} < \frac{1}{x} \). (Take as given that \( x \) and \( y \) are elements
of a given ordered field \( F \).)

(2) Write down the negation of each of the following statements.
(a) \( \forall \varepsilon > 0, x \in A, \exists y \in B \) such that \( d(x, y) < \varepsilon \). (Take as given that \( A 
\) and \( B \) are subsets of some metric space \((X, d)\).)
(b) \( \forall \varepsilon > 0 \exists \delta > 0 \forall f \in F, x \in X, y \in X \left( d_1(x, y) < \delta \implies 
(d_2(f(x), f(y)) < \varepsilon) \right) \). (Take as given that \( F \) is a set of functions
whose domain is the metric space \((X, d_1)\) and whose range is another
metric space \((Y, d_2)\).)

(3) Construct the truth table for the statement \((P \wedge Q) \implies (P \vee Q)\).
(4) TeX up one of the solutions (your choice) to your regular 100B/C problem
set from the assignment due Friday, September 25. (This will be part of
every future recitation homework assignment.)