

Values of analytic functions at algebraic points

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1. Content. Gwyen began by defining algebraic and transcendental numbers: $z \in \mathbb{C}$ is called *algebraic* if it is a root of a polynomial with integer coefficients; otherwise, z is called *transcendental*. She let \mathbb{A} denote the set of all algebraic numbers, and she noted that \mathbb{A} is a subfield of \mathbb{C} ; furthermore, \mathbb{A} is algebraically closed.

Gwyen continued by letting $\mathbb{C}\{z\}$ denote the set of Laurent series $f = \sum_{r=r_0}^{\infty} f_r z^r$ where r_0 is an integer, $f_r \in \mathbb{C}$, and f converges in $U_\rho := \{z \mid 0 < |z| < \rho\}$ for some ρ positive or infinity. Next, Gwyen introduced another concept: a series f is called *algebraic* if there exists a polynomial $A(z, w)$ with coefficients in \mathbb{C} such that $A(z, f) = 0$ formally. If so, then the analytic function $f(z)$ on U_ρ satisfies the equation $A(z, f(z)) = 0$, and $f(z)$ too is called *algebraic*. If there is no such polynomial $A(z, w)$, then f and $f(z)$ are called *transcendental*.

Gwyen made one more definition before discussing some theorems. She let S_f denote the set of algebraic points z in U_ρ such that $f(z)$ is algebraic. Next, she asked an interesting question: for which f is $S_f = \mathbb{A} \cap U_\rho$? Then she gave the first theorem.

Theorem 1-1. *If f is algebraic and its minimal polynomial $A(z, w)$ has algebraic coefficients, then $S_f = \mathbb{A} \cap U_\rho$.*

After proving this theorem, Gwyen concluded her talk by stating a theorem of Faber's.

Theorem 1-2. *There exists a transcendental analytic function $f(z) = \sum_{r=0}^{\infty} f_r z^r$ for which the f_r are integers and $S_f = \mathbb{A} \cap U_\rho$.*

2. Delivery. Gwyen's lecture was excellent overall, and only minor improvement is needed. She did a good job of engaging her audience. The mathematics was well organized, and the material flowed nicely; for the most part, the presentation had an appropriate level of detail. Unfortunately, Gwyen seemed a little nervous at times, especially at the beginning, but nervousness can be overcome with practice.

Gwyen did a superb job of motivating the definition of algebraic function. Still, her pace was a bit fast at first, though comfortable by the time she came to the theorems; I felt I could follow the proofs, yet was never bored. I suggest she spend more time discussing the background material before moving on to the main points. Also, she should give some examples, and say something about future directions.

Gwyen made good use of the blackboard, and had no particular problem. Notably, she did not erase her most recent material, and she put a panel up after writing on it. Usually, she wrote legibly. Important material was clearly displayed and labeled. She used abbreviations when she could. However, a few times, she omitted too many words from her written sentences, and they were a bit confusing.

Gwyen did a good job, by and large, of projecting her voice and speaking clearly, but there were times when she did not complete her sentences. Also, she didn't always say out loud what she was writing. Finally, she made little eye contact with half her audience.