Scrambled Proof #1

Consider the following theorem and the “pieces” of the proof you’ve been given. Please put the pieces in logical order to form an understandable proof.

**Theorem:** Let $x, y \in \mathbb{Z}$ and assume $x$ is even and $y$ is odd. Then $x + y$ is odd and $x \cdot y$ is even.

**Proof:**
1. \(2(c(2c' + 1)) = 2(2cc' + c)\)

2. Finally, we conclude that \(x\) even and \(y\) odd implies \(x + y\) is odd and \(x \cdot y\) is even, as desired.

3. \(x + y = (2c) + (2c' + 1)\)

4. Thus, by definition of odd, we know \(x + y = 2(c + c') + 1\) is an odd integer.

5. Thus, by definition of even, we know that \(x \cdot y = 2(2cc' + c)\) is an even integer.

6. \((2c)(2c' + 1) = 2(c(2c' + 1))\)

7. This is because there exists an integer, in this case \(k = (c + c')\), such that \(x + y = 2k + 1\).

8. There exists \(c \in \mathbb{Z}\) such that \(x = 2c\).

9. Next, we consider \(x \cdot y = (2c)(2c' + 1)\)

10. Assume \(x, y \in \mathbb{Z}\).

11. \((2c + 2c') + 1 = 2(c + c') + 1\)

12. \((2c) + (2c' + 1) = (2c + 2c') + 1\)

13. This is because there exists an integer, in this case \(j = (2cc' + c)\), such that \(x \cdot y = 2j\).

14. Assume \(x\) is even and \(y\) is odd.

15. There exists \(c' \in \mathbb{Z}\) such that \(y = 2c' + 1\).
Key: 10, 14, 8, 15, 3, 12, 11, 4, 7, 9, 6, 1, 5, 13, 2

Note #1: Statements 8 and 15 are interchangeable without altering the logic of the proof.

Note #2: Two portions of the proof are interchangeable without altering the logic of the proof. One could prove the statement about $x \cdot y$ before proving the statement about $x + y$. 