Score:* _____ *out of 10 **Name:** ____________________

MATH 221 – Foundations of Mathematics – Dr. Russell E. Goodman
Fall 2010 – Take-Home Quiz #8

This quiz is worth 10 points. Consider the theorem stated below along with the 12 sentences provided on the second page. Rearrange the sentences so the resulting text forms a logical and well-organized proof of the theorem. Please ***literally*** *cut out the 12 sentences and tape them to this first page in the order you feel is appropriate and turn in the taped quiz* ***by 4pm on December 8****.*

You are expected to work alone on this quiz, neither giving nor receiving assistance to/from any classmates, friends or professors. Any violation of this expectation will result in a grade of zero on this quiz for all involved parties.

Theorem: *Every integer greater than 1 is expressible as a product of primes.*

1. Hence, we see that $n = ((p_1 \cdot p_2 \cdots p_s) \cdot (q_1 \cdot q_2 \cdots q_t))$.

2. But a and b must then be expressible as products of primes due to the minimality of n.

3. It also must be the case that $1 < b < n$.

4. Then n must be a composite number (non-prime).

5. Suppose there are some positive integers greater than 1 that are not expressible as products of primes.

6. Thus, $a = p_1 \cdot p_2 \cdots p_s$ and $b = q_1 \cdot q_2 \cdots q_t$ where all p_i and q_j are primes and $s, t \geq 1$.

7. By the Well-Ordering Principle, there must be a smallest such integer n.

8. Therefore, it must be the case that every integer greater than 1 is expressible as a product of primes.

9. As a result, there must be an integer a such that $1 < a < n$ and such that $a|n$.

10. We proceed using proof by contradiction.

11. Using the definition of divisibility, there is an integer b with $a \cdot b = n$.

12. This is contrary to our assumption about n.