Theorem. For any \( n \in \mathbb{N} \),
\[
1 + 2 + \cdots + n = \frac{n(n + 1)}{2}
\]

Proof 1: We will prove the theorem by induction.
Base case: Since \( 1 = 1(1 + 1)/2 \), the theorem holds for \( n = 1 \).
Inductive step: Assume \( \sum_{k=1}^{n-1} k = \frac{(n-1)n}{2} \). Then
\[
1 + 2 + \cdots + (n - 1) + n = \frac{(n-1)n}{2} + n
= \frac{n(n-1)}{2} + \frac{2n}{2}
= \frac{n(n+1)}{2}.
\]

By induction, it follows that for every natural number \( n \),
\[
\sum_{k=1}^{n} k = \frac{n(n + 1)}{2}.
\]

Proof 2: We will prove this by reorganizing the terms of twice the sum on
the left hand side. We can reorganize the sum
\[
2(1 + 2 + \cdots + n) = (1 + 2 + \cdots + n) + (n + n - 1 + \cdots + 1)
= (n + 1 + \cdots + n + 1) = n(n + 1).
\]

Dividing by 2 gives the desired equality
\[
1 + 2 + \cdots + n = \frac{n(n + 1)}{2}.
\]