Thoughts About Teaching Communication in Mathematics Seminars
S. Ruff
Lecturer, Writing Across the Curriculum

This document was prepared for a workshop for mathematics seminar instructors at MIT. Mathematics seminars count toward the undergraduate communication requirement for majors in mathematics, so according to the Office of the Communication Requirement [1], each seminar should

- require at least 5000 words of writing including one mandatory revision, an equivalent amount of oral presentations, or an equivalent combination of the two;
- include substantial instruction and feedback on student work;
- integrate writing and speaking assignments that relate to the professional discourse in the major field; and
- count communication-intensive activities as a substantial portion of the final grade (>25%).

This document addresses issues faced by experienced mathematics instructors who find themselves teaching and grading communication in seminars. The suggestions that follow are based on 4 years of teaching technical communication in CI-Ms in Courses 18, 6, and 7, and from experience as a student in a Combinatorics seminar at Swarthmore College. Improvements were made thanks to comments from Steve Kleiman, Haynes Miller, David Roe, participants in past workshops, and others. This document has not been read or approved by the Subcommittee on the Communication Requirement at MIT.

This document starts by addressing common questions (about grading) and then briefly outlines other relevant issues.

Table of Contents
Grading 2
Characteristics of good mathematical communication 3
Grading revisited: grade sheets, weighting, and draft grades 5
Peer critiques 6
Commenting 6
Designing writing assignments 8
Lecturing about writing 8
Appropriate use of sources 8
Resources for seminar instructors 8
Acknowledgements 8
References 8

Appendices
A: Sample List of Characteristics of “Good Mathematical Writing” 9
B: Sample Grading Sheet from 6.021 10
C: Sample “Writing Quality” Grade Sheet from 3.155J/6.152J 12
D: Sample List of Coded Comments from 3.155J/6.152J 14
E: Dimensions of Commenting 16
F: Acceptable Use of Sources 17
G: Connectivity Example 18
Grading
How should communication in CI-Ms be graded? This question is difficult, so it’s helpful to identify objectives that can inform the answer. The CI requirement was designed in response to alumni who reported that they hadn’t received sufficient preparation for the communication required of them after MIT. So a reasonable objective for mathematics CI-Ms might be to prepare students for the communication that will be required of them if they go on in mathematics. For example, if mathematicians are required to convince people of the correctness of their arguments, then in seminars it may make sense for the grade to be based in part on how well students convince the instructor of the correctness of their arguments. If mathematicians are required to follow a certain structure when they write journal articles, then it may make sense for the grade to be based in part on how well students follow that structure. And so on. The following sections discuss various criteria of “good mathematical communication,” how to quantify “good” with a grade, and how comments can be phrased to most effectively help students learn “good mathematical communication.” First, though, there are a few misconceptions about CI-Ms and writing that should be addressed.

Misconception: Writing is too subjective to grade. Every grader is likely to assign a different grade.
Response: When writing instructors from Writing Across the Curriculum read and grade the same papers, their grades can agree to within a fraction of a letter grade if they have jointly practiced using a set of grading criteria. So it is possible to grade to the precision of a letter grade. Grading criteria are discussed in the next section.

Misconception: Quality of communication must count for at least 25% of the final grade.
Response: According to the Office of the Communication Requirement [1], CI-Ms should “count communication-intensive activities as a substantial portion of the final grade (> 25%).” Note that the requirement says “communication-intensive activities,” not “quality of communication.” In Course 6, although communication-intensive activities count for at least 25% of the final class grade, in some cases almost all of the grade for those communication-intensive activities is based on content.

Misconception: “Quality of communication” and “understanding of mathematical content” are independent, so they should be addressed separately.
Response: If “quality of communication” is grammar and mechanics, then it may be independent of the content; for example, the content may be clear even if the grammar is atrocious. However, if “quality of communication” includes such things as structure, flow, convincing argumentation, etc., the relationship between communication and content is not so simple. For example, if students don’t understand the content, their writing is likely to be imprecise or unclear; similarly, if students are just beginning to understand the content, they may not have the perspective necessary to effectively structure and frame their communication. For these students, communication and content grades are likely to be linked, and “quality of communication” is unlikely to improve much until “understanding of content” improves. (Interestingly, seeing and facing problems with their writing often prompts students to see and face their problems with the content.)
So CI-M instructors may choose to grade communication-intensive activities based on the content, on the “quality of communication,” or on both, but it’s important to realize that “quality of communication” means different things to different people. When an instructor identifies criteria against which to grade communication, those criteria should ideally be specific and well-understood by everyone who will be applying the criteria. The next section is intended to help instructors identify and prioritize criteria for grading communication-intensive activities.

**Characteristics of good mathematical communication**

Good communication has many different aspects, and different people are attuned to different aspects. For example, if two or more people read the same piece of writing, they are likely to notice different problems with the writing. One might immediately notice a misused technical term. Another may lament the repeated vague uses of “this” and “it.” Another might realize that the writing is disjointed and seems to have no clear point. In the technical writing industry, a rule of thumb is that 6 different readers are necessary to identify all of the errors in any given piece of text. Luckily, the job of the seminar instructor is not necessarily to help students to produce perfect text, but rather to help students to improve their writing skills in general. In can be helpful to realize that students will not become expert writers (or presenters) in one semester. As illustrated in Fig 1, becoming an expert writer, like becoming an expert in anything, takes thousands of hours of practice. So if an instructor can simply identify some important mistakes that the student tends to make repeatedly, and teach the student to avoid those mistakes, the student has learned and the instructor has succeeded.

![Graphs](image)

**Figure 1 Model of communication ability vs. practice time.** To become an expert at anything, including writing and speaking, takes thousands of hours of practice. Students will not become expert at communicating mathematics in one term. A seminar that simply helps students along the curve (a) or accelerates students’ learning (b) can be considered to have succeeded. An instructor may focus on only the most important issues, rather than trying to address all issues.

Focusing on the most important issues helps to reduce the complexity of teaching communication, but much complexity still remains. Which issues should an instructor focus on? How can an instructor grade fairly if no attempt is made to help the student achieve perfection during the term? If grades are based only on effort or improvement, how can students receive an honest assessment of where they are along the curve of communication ability (Fig 1)? The last two questions are addressed in a later section. The remainder of this section focuses on the first: identifying the criteria of good
mathematical communication and identifying which criteria are most “important” for each student. The rest of this document focuses primarily on writing, but a similar analysis can be done for presentations.

The bottom line is that it’s up to each instructor to determine which criteria he or she considers to be important. Some criteria and some general guidelines for determining “importance” are suggested below, and more can be identified in workshop with other instructors, but it is up to each instructor to determine which criteria to emphasize in his or her seminar. Different instructors are likely to make different choices, and instructors might want to alert students to the fact that communication instruction in the seminar is not exhaustive, and that students will have to continue to learn about other aspects of communication after the seminar ends.

To identify criteria of good mathematical writing, it’s instructive to look at the same piece of writing with several other experts in mathematical communication. (Seminar instructors are considered to be experts in mathematical communication because they have been immersed in the communication of the field for years.) As mentioned above, everyone is likely to notice different things in the writing sample that are done well and different things that are done poorly— together, a collection of criteria of good mathematical writing can be assembled. At first, the collection may appear to be a haphazard list. For the list to be useful, it’s helpful to organize the criteria. One organization is illustrated in Figure 2, which orders criteria roughly from “paper-level issues” to “word-level issues.” Two other organizations are illustrated in Appendix A. Both Figure 2 and Appendix A are intended only to give a sense of the variety of possible criteria, and of the fact that it’s possible to organize the criteria: neither is intended as a definitive list of criteria for good mathematical writing. It is up to instructors to construct their own lists.

“Paper-level issues”

Lack of understanding of the mathematics (may also be word-level)
Lack of focus/purpose to the paper
Unclear definition of audience
Poor paper structure: inappropriate division into sections &/or ordering of sections
Poor section structure: lack of focus/purpose to the section, poor ordering of information within the section
Poor audience awareness: (i.e. simply putting ideas on paper rather than trying to communicate to readers: neglecting to lay groundwork, guide readers through logic, etc)
For exposition: poor paragraph structure (e.g. unified paragraphs, topic & concluding sentences, connectivity between sentences and between paragraphs)
For exposition: sentence structure (convoluted wording, misplaced emphasis)
Word choice
Conventions of formatting, citation, grammar, etc

“Word-level issues”

Figure 2 A partial list of issues that interfere with good mathematical writing, in rough order from “paper-level issues” to “word-level issues.”
Grading revisited: grade sheets, weighting, and draft grades

Once a list of criteria has been assembled, as described in the preceding section, that list can be used to assign grades; the different criteria or groups of criteria can be weighted to reflect the instructor’s priorities. Appendices B and C show sample grade sheets used in 6.021 and 6.152. By using such grade sheets, instructors can not only grade consistently and transparently, but they can also ensure that all important criteria are considered for each paper, and they can train multiple graders to give consistent grades. Because each seminar is different and different instructors have different priorities and different ways of thinking about writing, each instructor is likely to find most helpful a grade sheet that he or she has created.

Earlier the following two questions were deferred:

- **How can an instructor grade fairly, given the realization that no attempt is being made to help the student achieve perfection during the term?**

- **If grades are based only on effort or improvement, how can students receive an honest assessment of where they are along the curve of communication ability (Fig 1)?**

The grade sheet in Appendix C illustrates one way to address the first question. The instructor who created the grade sheet prioritizes paper-level and paragraph-level issues more highly than the word-level issues of sentence structure and word choice: the first two categories are weighted at 30% each, while the last category is weighted at only 20%. Notice also that the definitions of the letter grades for the last category ensure that a student can receive an A(-) even if there are occasional errors in the text. The weighting and the grade definitions enable the instructor to help students primarily with the higher-priority issues without penalizing them for the lack of instruction on the lower-priority issues; yet students are still alerted to those less-important issues.

A solution to the second question is illustrated by the grade sheets in Appendices B & C. Notice that both grade sheets include “First Draft” grades. For the first draft, students receive a grade based on effort or completeness. The first draft is also assigned letter grades for the other criteria so students know how well they are meeting expectations, but those grades do not “count.” After students revise and hand in their final paper, the revised paper is used to determine new grades that “count.” In this way, draft grades are based on effort, and students still get a clear sense of how they’re doing.

Experience with giving first-draft “grades” that don’t count suggests that it’s helpful to make those grades slightly harsh on the first draft so that there’s room to improve the grade by a fraction of a letter grade even if a student makes only slight improvement: it’s very discouraging for students to put effort into a revision and receive no grade improvement as a result.

Experience also suggests that if first drafts grades do count and are based on quality rather than on effort/completeness, it’s wise to not weight those grades heavily. Because different readers focus on different issues, students are not likely to understand what’s important to their instructor until after they’ve received a round of feedback from him or her. Additionally, learning to write well requires some experimentation, and students are not likely to feel free to experiment if they’re too concerned about grades.
Peer critiques
Students can learn a great deal about clarity of communication by having classmates respond to their work and by responding to the writing of classmates. This experience also prepares students for communication after MIT, when they are unlikely to receive feedback unless they seek it from peers. However, students need guidance to write good critiques, so it’s helpful if instructors tell students what is expected and monitor and respond to the critiques, at least in the beginning of the semester. A grade can be helpful to provide motivation to the students to do a good job with the critiques. The grade sheet in Appendix B gives an example of how grades could be assigned to peer critiques.

Commenting
The wording and focus of comments affect how students respond to those comments: some comments help students to learn, others may encourage students to unthinkingly make the marked changes, and others may turn students against writing.

How best to comment depends on whether there is only one correct way to solve the problem (for example, when a student doesn’t appropriately use the conventions and language of the field) or whether there may be multiple successful ways to solve the problem (as when an explanation is difficult to follow).

If there is only one correct way to solve the problem, the instructor must tell the student what to do (or direct the student to an appropriate reference). However, if the instructor marks all instances of the problem, the student is likely to make the changes without thinking and is likely to make the same mistake again in the future. An alternative is to mark the problem once, tell the student to find and fix all other instances, and tell the student to add the problem to their editing checklist. (An editing checklist is a checklist of common mistakes that each student can make for him or herself and should go through before handing in papers.) It can be discouraging for instructors when students make the same errors over and over again, but it may be helpful to realize that learning the conventions and language of a field is like learning a foreign language: it can be achieved through a combination of immersion (listening and reading) and direct instruction (comments), but it takes years to become fluent in the language.

Although some writing problems require the instructor to tell the student exactly what to do, many problems with exposition have more than one possible solution. When text is confusing or unclear, often an instructor can help the student to improve the text simply by pointing out the problem: “I don’t understand this sentence. Are you saying _____ or are you saying _____?” “I’m not yet convinced that this is true. What if…” The instructor could suggest a solution or two, but ideally the students should choose their own solutions. The solutions a writer chooses are what comprise that writer’s “style.” Also, if the student is told what to do, the student is likely to make the corrections without thinking about them and is less likely to learn than if he or she chooses or comes up with the solution. Another problem with telling the student what to do is that instructors don’t always fully understand what students are trying to accomplish, so students are occasionally able to identify better solutions than those proposed by the instructor. So,
when possible, it’s beneficial to students if instructors point out problems but give
students some leeway to choose or find their own solutions.

Should an instructor point out all of the different kinds of problems with a paper, from
paper-level issues down to word-level issues? Some students appreciate receiving a large
amount of feedback, while others are overwhelmed by the comments. In either case,
students often need guidance to prioritize the comments. For those instructors who are
inclined to give many comments, there are various ways to indicate the relative
importance of those comments. One way is to emphasize the most important comments in
a summary note or on a grade sheet, as illustrated in Appendix C. Another option is to de-emphasize the less-important comments by using a coded list of common comments like
that in Appendix D.

Whether an instructor chooses to give many comments or few, which comments should
the instructor emphasize? If students do only one writing assignment with one revision
(as with an end of term paper), then there is time to emphasize only one set of issues, so
the instructor should choose whichever issues she or he considers to be most important.
But if students do multiple writing assignments or revisions during the term, then the
instructor has the leeway to emphasize different issues for each assignment or revision. In
this case, it may be most efficient to emphasize paper-level and paragraph-level issues
before emphasizing word-level issues. (See Fig. 2 on p. 4 for examples of paper-level vs.
word-level issues.) If students polish wording before larger-scale problems are addressed,
some polished text is likely to need to be discarded or rewritten, and the revision is likely
to require additional polishing. Although discarding text is a normal part of writing, it’s
possible to minimize the amount of text that’s discarded by focusing on larger-scale
issues earlier in the writing process.

Identifying paper-level and paragraph-level issues is challenging. Most instructors find
that larger-scale issues are obscured by the smaller-scale issues, and don’t become visible
until those smaller-scale issues are fixed. It is possible, though, to learn to see large-scale
issues even while smaller-scale problems are present. A grading sheet that includes large-scale issues can be helpful as a reminder to look for them.

Students often take comments about their writing personally, and occasionally comments
can turn students against writing. Students are more likely to be receptive to comments if
criticisms are offset by occasional honest praise. Because instructors may not be in the
habit of giving praise, it may initially be necessary to consciously look for honest praise
to give, but experience shows that honest praise is an effective motivator. To prevent
comments from being taken too personally, the comments should be worded to describe
the text, not the student: “this paragraph should be more concise” not “you don’t write
concisely enough.” Reading papers can be frustrating, so instructors must take care to
remain professional when frustrated.

Appendix E summarizes some of the issues involved in writing helpful comments for
engineering papers; many of the issues apply also to mathematics papers.
Designing writing assignments
For students to write well, assignments must be carefully designed. Some common pitfalls include the following:
*If the purpose or audience isn’t clear, the writing is likely to be unfocused.
*If the assignment is “Write about ________” the paper is likely to be difficult to structure well.

Lecturing about writing
Students report that they learn writing best from one-on-one meetings about their writing and from comments on their papers. Writing lectures are less effective and should probably be limited to giving background specific to an assignment or to addressing issues that show up in the writing of all of the students.

Appropriate use of sources
Depending on the assignment, students may rely overly heavily on one source and instructors may need to encourage them to find and use other sources. Appendix F can be handed out to students to demonstrate how to cite sources and how do indicate the degree to which the source has been used.

Resources for seminar instructors
When a reader has a hard time reading or focusing on exposition but doesn’t know why, there’s often a structural problem or paragraph-level problem with the text. If a student paper exhibits this problem, the student may benefit from reading the article “The Science of Scientific Writing” by Gopen and Swan [2]. This article is also helpful to many intermediate and advanced writers. A Google search generates many hits.

If students have trouble with exposition, feel free to refer them to Susan Ruff (ruff@mit.edu). Appendix G gives one example of the sorts of issues Susan can help students address.

The Writing Center may also be a helpful resource: web.mit.edu/writing
If no appointments are available at the Writing Center, students can sign up on the waiting list; students on the waiting list very often wind up with appointments.

Many seminar instructors have been teaching mathematical communication for years. Peers are often the best resource.

Acknowledgements
Many thanks to Steve Kleiman, from whom I learned much of what I know about mathematical communication; Dave Custer, who developed the model illustrated in Figure 1; Denny Freeman, Mya Poe, and Jongyoon Han, who developed the grade sheet in Appendix B; and the above, Haynes Miller, David Roe, and others, who offered helpful comments.

References
Appendix A: Sample Lists of Characteristics of “Good Mathematical Writing”

These lists are intended only to give a sense of the variety of characteristics that are involved in good mathematical writing, and of the fact that it’s possible to organize the characteristics into groupings (although, regardless of grouping, there are likely to be some items that seem to belong in none of the groups or in more than one of the groups). Similar lists can be created for presentations. These lists are not intended to be definitive: seminar instructors should ideally create their own lists.

Sample List 1

Mathematical correctness & logic
- Mathematical content is correct.
- The paper is broken logically into sections
- Each section follows a logical development.

Development
- The stage is set before details are given—in the introduction, at the beginnings of sections, and wherever needed throughout explanations.
- The reader is guided through the logical development of arguments.
- Explanations are appropriate for the target audience: difficult content is presented in such a way that readers must think, but are rewarded by being able to follow the logic.
- Appropriate examples are provided as needed to clarify explanations.
- Figures and tables successfully illustrate &/or clarify important points of the text, as needed.
- Sources are used appropriately to support the text, and there isn’t an over-reliance on a single source
- In exposition, tools such as paragraphs, concluding sentences, information order, etc, are used as needed to present the content clearly.

Conventions of structure, wording, notation, grammar, etc.
- Notation is used appropriately.
- The wording and tone are appropriate for a mathematics paper.
- The wording and tone are appropriate for the assigned context and audience.
- The structure of the paper and of each section satisfies conventions for the structure of mathematical papers.
- Conventions for formatting and referencing figures and tables are followed.
- Conventions of citation are followed.
- Grammar is correct.
- Punctuation.

Sample List 2

This list outlines the structure of Steve Kleiman’s, “Writing a Math Phase Two Paper,” which describes characteristics of good mathematical writing for those who wish to publish in MIT’s Undergraduate Journal of Mathematics.

Organization
- Parts of a paper (title, abstract, introduction, etc.)
  --includes sectioning and emphasizing important points

Language
- Precision
- Clarity
- Familiarity, Forthrightness, Conciseness, Fluidity, Imagery

Mathematics
- Formulas
- Assertions (conjectures, theorems, corollaries, etc.)
- Proofs
- Symbols, punctuation, and notation

See “Writing a Math Phase Two Paper” for more information about each of these categories.
Appendix B: Sample Grading Sheet from 6.021

Name(s):

Technical Reviewer:

Writing Reviewer:

First draft (10%).
A: Complete first draft.
B: Significant work, but paper needs significant revision.
C: Incomplete descriptions, missing sections.
D: Few results, few graphs, few discussion points.

Critique (5%).
A: Several helpful high-level suggestions (e.g., suggesting major restructuring, new figures, ...) plus probing questions (could your result be caused by...?), plus appropriate low-level comments (e.g., grammar).
B: At least one helpful high-level suggestion or probing question plus appropriate low-level comments.
C: Helpful low-level comments.
D: Few helpful comments.

Clarity and Conciseness of Exposition (15%).
A: Content of each paragraph is readable with clear, simple prose and appropriate use of technical language. Each graph or table clearly supports the prose and has a meaningful title and/or caption.
B: Content of report is readable with minor slips in clarity or a single unclear section. Some technical language may be inaccurate but does not impair meaning. Graphs and tables have functional titles and/or captions.
C: Repeated wordiness. Language is too informal or inaccurate for scientific writing. Descriptions of graphs and tables are weakly developed. Proofreading errors.
D: Accumulation of stylistic errors that seriously interfere with report readability and/or missing figure titles and captions which make it difficult to link prose to data. Numerous proofreading errors.
**Clarity and Conciseness of Technical Information (20%)**
A: Technical flow is clear; introduction motivates a topic, results focus on that topic, plots support the argument in the text, conclusions follow from results, relevant methods are described. No extraneous materials.
B: Results are clear, plots support the argument in the text, conclusions follow from the results. No extraneous materials.
C: Results are clear, plots support the argument in the text.
D: Results are not clear or plots do not support the argument in the text.

**Technical Content (25%).**
A: All sections are clear, results report more than one significant technical finding or conclusion, and there are no technical errors.
B: All sections are clear, results contain at least one significant technical finding or conclusion, and there are no technical errors.
C: All sections are clear and technical findings are described without any major technical errors.
D: Major technical errors or too little technical content or too poorly written to assess technical content.

**Insightfulness (25%).**
A: Significant result that would be of interest to students in this class.
B: Clever experimental design or imaginative analysis.
C: Standard experiment that is executed well.
D: Not well executed or too poorly written to assess insightfulness.

**Weighted Average Grade:**

**Lateness Factor:**

**Grade:**
Appendix C: Sample “Writing Quality” Grade Sheet from 3.155J/6.152J

Name [name removed] First Draft Grade 3

Writing Instructor Susan Ruff, ruff@mit.edu

* Only the first draft grade below counts. The other three will change based on your revision, and are given here only to let you know how you are doing. If you were to hand in this draft as your final paper, these are the grades you would receive. Percents indicate percent of the total writing grade.

First draft (20%)
3/3: The draft is complete, incorporates feedback on the IC letter, is proofread, and is received on time.
2/3: The draft is not complete, not proofread, late, or shows minimal attention to feedback on the IC letter.
1/3: Multiple problems interfere with giving meaningful suggestions for improvement.

3 Complete, professional, and concise.

Large-scale comments (paper structure, section structure, argumentation, tables, figures) (30%)
A: Information appears in the appropriate sections. Each section has a logical structure that emphasizes important information; the structure helps readers to find information and to follow the logical flow of arguments, as needed. The discussion emphasizes the mechanical characteristics and supports contentions with quantitative evidence. Tables and figures present important information clearly.
B: Overall structure and organization are understandable, but could be improved in one section of the report or in minor instances throughout the report.
C: Unclear structure in more than one section of the report or repeated organizational problems interfere with report coherence.

(B-/C+) This paper was a pleasure to read. Your analysis is very interesting. I am not yet completely convinced by your analysis: further evidence &/or explanation would be helpful. See comments and questions on the discussion section. If the goal is to determine the mechanical characteristics of the film, what does your analysis say about which of your Young's moduli is most accurate?

The structure of the paper and each section is clear and appropriate in general, but there is an important problem: Figures 2 and 3 show results and should be introduced in the results section, not in the discussion section. I would also include Fig 4 in the results section, although it might be possible to argue that Fig 4 belongs in the discussion. You may still discuss the figures in the discussion section.

Additionally, the purpose of the paper isn't clear—the purpose isn't presented consistently throughout the paper. I suggest that you use the following as your unifying purpose: identify the most reliable value of E that you can, given your data. (If you decide to use this purpose, you would need to obtain Leslie's permission to ignore residual stress.)

On the paper itself, I've asked some questions that indicate potential points to add to the discussion. If you need more room, you could condense the conclusion and you could consider omitting Fig 1. I'd be happy to meet with you to identify other ways to make room if necessary.
Paragraph-level comments (30%)
A: Paragraphs are easy to read and understand and are structured to emphasize important information. Good information order and connectivity help readers follow the logical flow of the paragraph.
B: Paragraphs are understandable, but there are occasional problems with paragraph structure &/or connectivity.
C: Repeated problems with paragraph structure &/or connectivity interfere with report readability.

(A–) In general, the paragraphs are structured well and important information is emphasized appropriately. See comments on the paper itself. Good job.

Sentence Structure and Word Choice (20%)
A: Exposition is well-written at the word and sentence level with appropriate grammar, wording, use of articles, verb tenses, clear antecedents, level of conciseness, etc. (with rare exceptions)
B: Exposition is easily understood, but there are recurring sentence and word-level problems.
C: Sentence and word-level problems interfere with report readability &/or text is excessively wordy and exceeds the length limit.

(A-) The text is clear and professional in general, but there are rare problems at the sentence & word level. See specific comments and underlined text on the paper itself.

Let me know if you have any questions. I’d be happy to meet with you to go over these comments or to answer any questions you have as you revise.

--Susan
ruff@mit.edu
Appendix D: Sample List of Coded Comments from 3.155J/6.152J
Codes are used for less-important comments: those that students should know about but that shouldn’t distract them from the larger issues with their writing. The more important comments can be written directly on the paper for emphasis.

Your IC Letter may have comment codes on it. Refer to this list to find the meanings of the codes. You may also find it helpful to read through the whole list. Let us know if you have any questions.
—Susan (ruff@mit.edu) and Harlan (breindel@mit.edu), 3.155/6.152 Writing Instructors

F Figures, Tables, and data in text
F1 Make graphs as uncluttered as possible to focus attention on your results. Delete the border, background color, and gridlines.
F2 Unless there’s a reason to include the origin, adjust the axes to focus on your data.
F3 Be sure graph labels are large enough to be read easily.
F4 The graph title belongs in the figure caption. Putting a title both above the graph and in the caption is redundant; you don’t need to put a title above the graph.
F5 If you take a figure from another source, cite the source in the figure caption.
F6 Every figure and table must be referred to in text. Refer to the table or figure by number, not position: “…as shown in Table 2,” not “…as shown in the table below.”
F7 The table number and title belong on top of the table. Tables and figures are numbered separately.
F8 Design tables and graphs to emphasize the points you want to make. For example, if you refer to the accumulation and depletion regions when you discuss a graph, label those regions on the graph.
F9 Present the calculated and measured results in a table for ease of reference and comparison. Values that you want the audience to compare should be placed next to each other and should use the same units.
F10 Abbreviate units (“10 meters” = “10 m”)
F11 Always include units on graph axes and in tables.
F12 When units follow numbers, put a space between the number and the unit: “10 m” not “10m”
F13 For decimals less than one, use an initial zero so the number will not be misread: “0.5 m” not “.5 m”
F14 Present figures and tables in the order in which they are mentioned in text. When possible, put each figure and table on the same page as the reference to it in text (ideally after the reference).
F15 Avoid starting a sentence with a number, symbol, etc. If you must start with a number, spell it out.
F16 Punctuate equations as though they are grammatical parts of the text.

P Paragraph structure
P1 Use unified paragraphs—each paragraph should be unified around a single topic.
P2 Use a topic sentence to introduce the topic of the paragraph and to preview the structure of the paragraph if necessary. Ideally, the topic sentence should convey content.
P3 Use a concluding sentence to clarify the main point of the paragraph, especially if the paragraph is long or contains many details. Otherwise you risk losing your audience in the details.
P4 To improve connectivity between sentences, use the beginning of each sentence to link back to previous information. Then use that “old” information to introduce important new information, which should go in the stress position at the end of the sentence. For more information, see “The Science of Scientific Writing,” by Gopen and Swan. A Google search will generate many hits.

W Word choice and grammar
W1 “This” should be followed by its antecedent. For example, “this etch.” Replace “it” by its antecedent.
W2 Don’t omit articles (a, an, the).
W3 The word “data” is plural: “data are” “datum is”
W4 “Since” may be ambiguous because it can refer to either time or cause & effect. “Because” is preferred.
W5 The words “important,” “vital,” “essential,” etc are often signs that information is missing. Rather than saying that something is important, give your audience the information they need to conclude for themselves that it’s important. (If it’s not really important, don’t say it is.)
W6 “In order to…” can often be written more concisely as “To…”
W7 Put the action in the verb. Use “The wafers were annealed…” not “Annealing was performed on the wafers…”

Thoughts About Teaching Communication in Mathematics Seminars © 2007 S. Ruff
A Abstract
A1 The abstract should briefly summarize the purpose of your work, the methods, the results, and the conclusions, in that order. Background is not necessary and should be saved for the introduction.
A2 Include a brief but quantitative summary of your results.

I Introduction
I1 Your audience is familiar with microelectronics processing technology. You don’t need to give basic background information.
I2 For the purposes of 3.155/6.152, it’s acceptable for your introduction to be only one paragraph long.

E Experiment
E1 It’s understood that the experiment section is chronological. You don’t need to repeat “then” and “next” throughout.
E2 You do not need to name the equipment you used unless the equipment is nonstandard, in which case a brief description may be more useful to your audience than a name. Remember that your audience is familiar with microelectronics processing technology.
E3 Instead of simply listing the steps you followed (for example, BOE, HMDS, photoresist application, postbake, backside etches, photoresist strip), explain what you did: “The front of the wafer was coated with photoresist to protect the oxide and polysilicon layers during the backside etch.”
E4 Group the steps of the experiment logically into paragraphs. Begin each paragraph with a topic sentence to provide context for the steps described in that paragraph. For example, what was the end goal? (backside etch, patterning, characterization, etc)
E5 Use consistent verb tenses. The Experiment section is usually written in past tense.
E6 If you use subheads, use meaningful subheads. “Lab Session II” is not very informative; “backside etch” is more meaningful.
E7 Write specifically about your work. Avoid general wording like “Resistivity can be used to calculate...” Instead, indicate what you did: “Resistivity was used to calculate...”

R Results
R1 The Results section must contain text. Introduce the tables and figures in the text. You may also want to give an objective summary of your results. For example, “most of the values were within 5% of expectations, except...”
R2 Be quantitative. For example, if you say two values are different (or similar), give a % difference.

D Discussion
D1 Consider beginning the Discussion with an overview paragraph. What are the most important discussion points?
D2 Do not refer to the appendices in the Letter. A Letter normally does not have appendices. Results that the audience needs in order to follow the discussion should appear in the body of the Letter.

C Conclusion
C1 Summarize the most important points from the discussion. For example, “Most parameters were within 5% of expected values except…, which may have been caused by…”
C2 Nothing new should appear in the conclusion (except, perhaps, recommendations for future work). Any points that are important enough to be mentioned in the conclusion should also be discussed in the Results and Discussion section.
C3 Avoid vague words like “successful.” What exactly do you mean by successful? Be quantitative.
**Appendix E: Dimensions of Commenting**

Many issues are involved in writing good comments. Some of those issues are summarized below as different “axes” of commenting. The commenting extremes are shown for each axis, but no attempt is made to indicate where comments should fall along each axis: that judgment is a point for discussion. The axes are not orthogonal and the list is not exhaustive. Some examples are based on an article about commenting by Kerry Walk of the Harvard Writing Project; others are based on my experience.

### Personal

<table>
<thead>
<tr>
<th>Comments focus on student.</th>
<th>Comments focus on writing.</th>
</tr>
</thead>
<tbody>
<tr>
<td>“You write clearly.”</td>
<td>“This paper is written clearly.”</td>
</tr>
<tr>
<td>“I’m disappointed (in you.)”</td>
<td>“This paper isn’t as focused as your first.”</td>
</tr>
</tbody>
</table>

### Directive

<table>
<thead>
<tr>
<th>Comments tell students</th>
<th>Comments point out</th>
</tr>
</thead>
<tbody>
<tr>
<td>what changes to make.</td>
<td>problems and give</td>
</tr>
<tr>
<td></td>
<td>suggestions for</td>
</tr>
<tr>
<td></td>
<td>addressing them.</td>
</tr>
</tbody>
</table>

### Form vs. Function

<table>
<thead>
<tr>
<th>Comments focus on form</th>
<th>Comments focus on content.</th>
</tr>
</thead>
<tbody>
<tr>
<td>“This paragraph needs a topic sentence.”</td>
<td>“The main point of this paragraph isn’t clear.”</td>
</tr>
<tr>
<td>“The discussion is too short.”</td>
<td>“You need to support these claims with evidence.”</td>
</tr>
</tbody>
</table>

### General/vague vs. Specific

<table>
<thead>
<tr>
<th>“Your style needs”</th>
<th>“Vary the lengths”</th>
<th>“Combine the sentences as shown.”</th>
</tr>
</thead>
<tbody>
<tr>
<td>“some improvement.”</td>
<td>“of the sentences.”</td>
<td></td>
</tr>
</tbody>
</table>

### Scale (e.g. large-scale structural issues, small-scale wording issues):

<table>
<thead>
<tr>
<th>Comments focus on issues at only one scale.</th>
<th>Comments focus on issues at a variety of scales.</th>
</tr>
</thead>
</table>

### Focus and number

<table>
<thead>
<tr>
<th>Comments address all problems. (Many comments per paragraph.)</th>
<th>Comments focus on the few most important issues. If the paper needs major revision, comments focus primarily on the relevant large-scale issues, not on details.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 comment per paper</td>
</tr>
</tbody>
</table>

### Organization

<table>
<thead>
<tr>
<th>All comments are presented identically, regardless of importance.</th>
<th>Important comments are emphasized, for example by being included in a summary or written in a different color.</th>
</tr>
</thead>
</table>

Other dimensions include pos./neg. feedback, tone (collegial vs patronizing), & legibility.
Appendix F: Acceptable Use of Sources

State your main sources in the introduction. Clearly indicate what information comes from your sources and what information is original. Some examples follow; all examples are taken from the 2002 Undergraduate Journal of Mathematics, with minor editorial modifications.

The result treated in this paper was proved first by Kakutani in [4]. Intuitively, it states this: a continuous function \( f \) defined from a convex subset of an Euclidean space to the set of convex subsets of that set has at least one fixed point. The sense in which \( f \) is continuous and has a fixed point is formalized in Section 2...

...Section 2 below gives the basic definitions, and reviews a proof of Kakutani’s Theorem suggested by Hongu He (priv. comm., June 1998). Section 3 proves a generalization, due to the author, in which the original continuity assumption is relaxed.

—C. Chiscanu, *Fixed-Point Theorems*

In Section 2, we discuss the neoclassical, monopsony and revisionists’ models characterizing economists’ views on the minimum-wage controversy. In Section 3, using these models, we analyze the debate spurred by Card and Krueger [1]. In Section 4, we assess Card and Krueger’s assertions. Finally, in Section 5, in a replication and extension study, we strengthen our conclusions with empirical evidence.

—M. Fernandez, *The Minimum-Wage Controversy*

The writing in your paper should be your own, so you should try to avoid using wording from your sources. When possible, paraphrase and cite your sources. To paraphrase, understand what your sources are saying, then put your sources away before writing. It’s fine if the resulting explanation is not as elegant as the explanations in your sources, as long as the explanation is correct. If you paraphrase, you must still cite your sources, unless the information you are summarizing is considered “common knowledge.” If a whole section is essentially a paraphrase or summary based on your source(s), place the citation near the beginning of the section. Remember to include the page numbers.

Occasionally, to be accurate and precise, you must copy wording exactly from a source. In that case, clearly state that the text is taken from your source, and include a citation.

Now that we have in intuitive picture, we present a formal statement and proof of Menger’s theorem. This proof comes from West [2, pp. 149–51]. There exist more than fifteen different proofs of Menger’s theorem. West’s proof, although not the shortest, is more intuitive than others. We follow it with some slight modifications.

—R. Miller, *Introduction to Network Theory*

Also cite statistics and other facts that are not common knowledge.

When the Internet was originally designed during the 1960s, it was done as a computer science project at several universities, and was sponsored by DARPA, the Defense Advanced Research Project Administration [1].

—R. Miller, *Introduction to Network Theory*
Appendix G: Connectivity Example
The following text is a paragraph taken from the middle of a student paper.

Original paragraph:
The permutation groups are presented to us by a set of generators. We first implement basic operations on permutations such as composition and inversion. We then implement the Stripping algorithm of Sims to replace the original given set of generators by a small ‘stripped’ set (that generates the same group) which will ensure efficient computation of other properties of the group structure. Using the orbit structure of the group and Schreier’s Theorem, we present an algorithm which computes the order of the permutation group.

This paragraph doesn’t flow as well as it could. Ideally, each sentence should be logically connected to the preceding sentence, and the logical connection should be clear to the target audience. In this paragraph, the connection between the first two sentences isn’t clear: the missing information is that the reason for using composition and inversion is to generate the permutation group. The connection between the last sentence and the preceding sentences is also not clear. Finally, the third sentence tries to present too much information, so the relationship between the different parts of the sentence isn’t clear. To someone familiar with the content, the lack of connectivity is not an issue. But to someone new to the content, this paragraph is difficult to read.

To sort out the paragraph, it’s necessary to first identify the point of the paragraph. For the sake of this analysis, assume that the point of the paragraph is that a stripped set of generators can be created, and that this stripped set can be used to obtain information about the group. Once the point of the paragraph is identified, the paragraph can be structured in such a way that it builds to and supports that point. (BTW, the following revision mentions S, X, and G. These were introduced in the preceding paragraph in the paper, so at this point they are familiar to the audience.)

Revised text:
We generate the permutation group G from S by implementing basic operations on permutations such as composition and inversion. The same group G can be generated by a variety of different sets S; we can identify a “stripped” set S by implementing the Stripping algorithm of Sims. This stripped set can be used to efficiently compute other properties of the group structure. For example, the stripped set of generators for the stabilizer of an element X can be used to compute the order of G.

It’s difficult for authors to identify problems with connectivity on their own, because the ideas are not new to the author, so the author is unlikely to notice when the concepts aren’t explained clearly. One way an author can identify problem text is by checking to be sure that each sentence connects clearly to the preceding sentences.

For more information about connectivity, see “The Science of Scientific Writing,” by Gopen & Swan. Published in American Scientist 78(6) 550-558, November-December 1990 and available online.