RECITATION WORKSHEET: ORDER OF QUANTIFIERS

18.100C

A *sequence of real numbers* is a function $a : \mathbf{N} \to \mathbf{R}$; by convention, the value $a(n)$ is denoted a_n and the sequence itself is often denoted $(a_n)_{n\in\mathbf{N}}$.

Following is a list of six formal statements, (a)-(f), about a sequence of real numbers $(a_n)_{n\in\mathbf{N}}$, along with six example sequences (1)-(6). For each of (a) through (f), determine which if any of the sequences (1) through (6) satisfy the given property. Then describe in informal (English) language what each property means.

Properties of a sequence a_n

(a) $\forall \epsilon > 0 \; \exists N \in \mathbb{N} \; \forall n \geq N \; |a_n| < \epsilon$

(b) $\forall \epsilon > 0 \; \forall N \in \mathbb{N} \; \forall n \geq N \; |a_n| < \epsilon$

(c) $\exists N \in \mathbb{N} \; \forall n \geq N \; \forall \epsilon > 0 \; |a_n| < \epsilon$

(d) $\forall N \in \mathbb{N} \; \exists n \geq N \; \forall \epsilon > 0 \; |a_n| < \epsilon$

(e) $\forall N \in \mathbb{N} \; \exists n \geq N \; \exists \epsilon > 0 \; |a_n| < \epsilon$

(f) $\exists \epsilon > 0 \; \forall N \in \mathbb{N} \; \exists n \geq N \; |a_n| < \epsilon$

Sequences a_n

(1) $a_n = 0$ if n is prime, $a_n = n$ otherwise.

(2) $a_n = 0$

(3) $a_n = 1/(n+1)$

(4) $a_0 = a_1 = a_2 = a_3 = 3$, $a_n = 0$ for $n \geq 4$

(5) $a_n = \frac{\sqrt{2}}{2} + \sin(\pi n/100)$

(6) $a_n = 10^{10^n}$

Challenge Problem. Let S be a set and let $\mathcal{P}(S)$ denote the power set of S, i.e. the collection of subsets of S. Can you construct a bijection $f : S \to \mathcal{P}(S)$? What does this tell you about the cardinality of $\mathcal{P}(S)$?

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