

## Paraphrasing in a Mathematics Term Paper

The following examples illustrate how to use and acknowledge sources when writing your term paper. They are modeled on and should accompany *Academic Integrity: A Handbook for Students* [1, pp. 12-14].

### Examples: Plagiarism versus Paraphrasing

Original source [2]:

**Theorem 1.** *Let  $G$  be a finite graph with  $v$  vertices. Let  $C$  be a partial proper coloring of  $t$  vertices of  $G$  using  $d_0$  colors. Let  $p_{G,C}(\lambda)$  be the number of ways of completing this coloring using  $\lambda$  colors to obtain a proper coloring of  $G$ . Then,  $p_{G,C}(\lambda)$  is a monic polynomial (in  $\lambda$ ) with integer coefficients of degree  $v - t$  for  $l \geq d_0$ .*

**Proof.** We apply induction on the number of edges of the graph  $(G, C)$ . We consider three cases:

- (1) Let us suppose that  $e$  is an edge connecting two vertices of  $G$  at most one of which is contained in  $C$ ...
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Plagiarism

**Theorem 1 [1, p. 709].** *Let  $G$  be a finite graph with  $v$  vertices. Let  $C$  be a partial proper coloring of  $t$  vertices of  $G$  using  $d_0$  colors. Let  $p_{G,C}(\lambda)$  be the number of ways of completing this coloring using  $\lambda$  colors to obtain a proper coloring of  $G$ . Then,  $p_{G,C}(\lambda)$  is a monic polynomial (in  $\lambda$ ) with integer coefficients of degree  $v - t$  for  $l \geq d_0$ .*

**Proof.** We use induction on the number of edges of the graph, considering three cases:

- (1) Suppose  $e$  is an edge between two vertices, at most one of which is contained in  $C$ ...

### Why is this plagiarism?

The theorem is restated exactly without signaling to the reader that the wording is from the source, and restating this theorem exactly is neither necessary nor desirable. Sometimes restating theorems exactly is necessary in order to be correct; however, that is not the case with this theorem, as is demonstrated by the following example. Also, the proofs are almost identical, with a few wording changes here and there.

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Paraphrase/Rewrite (Student work used with permission, with minor editorial modifications)

**Theorem 2 [1, p. 709]:** *Given an initial partial proper coloring of graph  $G$ , let  $m$  be the number of initially colored vertices of  $G$ ,  $n$  be the total number of vertices in  $G$ , and  $l$  be the number of colors initially used. Let  $p(\lambda)$  be the number of ways to complete this coloring using  $\lambda$  total colors,  $\lambda \geq l$ . Then  $p(\lambda)$  is a polynomial that satisfies the following conditions:*

1.  $p(\lambda)$  is a polynomial of degree  $n - m$ .
2. The coefficient of the highest-degree term in  $p(\lambda)$  is 1.
3. The remaining coefficients are all integers.

Herzberg and Murty present two proofs of this theorem: the more direct proof uses partially ordered sets and Mobius functions, while the less direct proof uses induction. We follow their proof by induction [1, p. 710], elaborating as needed.

**Proof:** We proceed via induction on the number of edges of  $G$ . For the base case, consider a graph  $G$  with 0 edges. To extend the partial coloring of  $G$ , all remaining vertices can be colored with any color, so  $p(\lambda) = \lambda^{n-m}$ , which satisfies the above conditions.

For the induction step, suppose we have a graph  $G$  and assume all graphs with fewer edges than  $G$  satisfy the conditions of Theorem 2. There are three cases to address:

1. There exists an edge  $e$  in  $G$  between vertices  $v_1$  and  $v_2$ , at most one of which is initially colored...

### **Why is this acceptable?**

The writer has tailored the theorem to fit its new context and audience by omitting the unnecessary subscripts  $C$  and  $0$ , by including the induction proof's base case, and by making the content less dense and more structured so it's easier for readers to absorb. Although the overall structure of the proof is similar to that in the source, this similarity is acknowledged explicitly in the text.

*Note:* When a theorem statement *must* be restated exactly for it to be correct, do not place the theorem statement in quotation marks: instead, use text to signal the extent to which you've used the wording or structure of the source, as is illustrated in the example above. You can signal that a theorem is restated exactly by using the wording "from" or "taken from" (e.g., "This insight is formalized in the following theorem from Rudin.") Although theorems and definitions may sometimes be restated exactly, proofs should always be presented in your own words.

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### ***What strategies can I use to paraphrase a proof?***

Read and understand the proof as best you can. Then set your source aside and try to prove the theorem yourself in your own words. If you become stuck, try to figure it out yourself or talk it through with a friend, as you would for a pset. If you remain stuck, look at the source just long enough to become unstuck, and then repeat the process. Once you can complete the proof on your own, check the source to ensure that your proof is correct. Correct any errors, but resist using the source to make your proof more elegant. If your proof is inelegant, look for your own ways to make it more elegant; but it's fine if your proof is not as elegant as that in the source, as long as it's correct. Although this process is time consuming and difficult, it will help you to identify and understand the proof's subtleties. Remember to cite your source, even when you paraphrase!

It's easiest to avoid plagiarism if your sources are reasonably well written but aimed at an audience that is different from your target audience: that is, you should be able to understand your source, but you should have some need to modify the explanations for your audience. You might consider basing your choice of term paper topic in part on the available sources.

### **References**

- [1] *Academic Integrity at the Massachusetts Institute of Technology: A Handbook for Students*, web.mit.edu/academicintegrity/handbook/handbook.pdf Visited October 2, 2010.
- [2] Herzberg, Agnes M., and M. Ram Murty. "Sudoku Squares and Chromatic Polynomials." *Notices of the AMS*. 54.6 (2007): 708-17.