# ***Scrambled Proof #1***

Consider the following theorem and the "pieces" of the proof you've been given. Please put the pieces in logical order to form an understandable proof.

**Theorem:** Let $x, y \in \mathbb{Z}$ and assume $x$ is even and $y$ is odd. Then $x + y$ is odd and $x \cdot y$ is even.

**Proof:**

1. $2(c(2c' + 1)) = 2(2cc' + c)$

2. Finally, we conclude that $x$ even and $y$ odd implies $x + y$ is odd and $x \cdot y$ is even, as desired.

3. $x + y = (2c) + (2c' + 1)$

4. Thus, by definition of *odd*, we know $x + y = 2(c + c') + 1$ is an odd integer.

5. Thus, by definition of *even*, we know that $x \cdot y = 2(2cc' + c)$ is an even integer.

6. $(2c)(2c' + 1) = 2(c(2c' + 1))$

7. This is because there exists an integer, in this case $k = (c + c')$, such that $x + y = 2k + 1$.

8. There exists $c \in \mathbb{Z}$ such that $x = 2c$.

9. Next, we consider $x \cdot y = (2c)(2c' + 1)$

10. Assume $x, y \in \mathbb{Z}$.

11. $(2c + 2c') + 1 = 2(c + c') + 1$

12. $(2c) + (2c' + 1) = (2c + 2c') + 1$

13. This is because there exists an integer, in this case $j = (2cc' + c)$, such that $x \cdot y = 2j$.

14. Assume $x$ is even and $y$ is odd.

15. There exists $c' \in \mathbb{Z}$ such that $y = 2c' + 1$.

**Key:** 10, 14, 8, 15, 3, 12, 11, 4, 7, 9, 6, 1, 5, 13, 2

**Note #1:** Statements 8 and 15 are interchangeable without altering the logic of the proof.

**Note #2:** Two portions of the proof are interchangeable without altering the logic of the proof. One could prove the statement about $x \cdot y$ before proving the statement about $x + y$.