Theorem. *For any $n \in \mathbb{N}$,*

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

Proof 1: We will prove the theorem by induction.

Base case: Since $1 = 1(1+1)/2$, the theorem holds for $n = 1$.

Inductive step: Assume $\sum_{k=1}^{n-1} k = \frac{(n-1)n}{2}$. Then

$$\begin{aligned} 1 + 2 + \cdots + (n-1) + n &= \frac{(n-1)n}{2} + n \\ &= \frac{n(n-1)}{2} + \frac{2n}{2} \\ &= \frac{n(n+1)}{2}. \end{aligned}$$

By induction, it follows that for every natural number n,

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

□

Proof 2: We will prove this by reorganizing the terms of twice the sum on the left hand side. We can reorganize the sum

$$\begin{aligned} 2(1 + 2 + \cdots + n) = \ & (1 + 2 + \cdots + n) + \\ & (n + n-1 + \cdots + 1) \\ = \ & (n+1 + n+1 + \cdots + n+1) \\ = \ & n(n+1). \end{aligned}$$

Dividing by 2 gives the desired equality

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

□